

# Triangle-Mesh Models, Their Generation, and Their Application in Image Scaling

(PhD Oral Exam)

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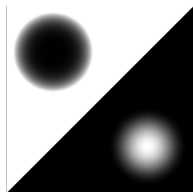
University  
of Victoria

- 1 Mesh generation with **minimal squared error** for image representation
  - Triangle-mesh models of images
  - ERD mesh model
  - Proposed SEMMG method and its development
  - Evaluation results

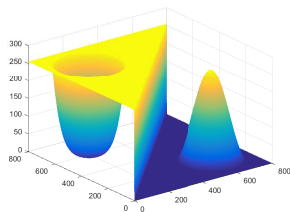
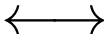
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- 3 Conclusions
- 4 Future research

# Triangle Mesh Models of Images

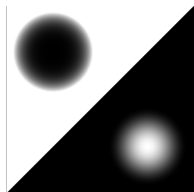


original image  $\phi$

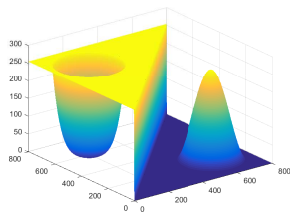
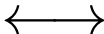


surface model

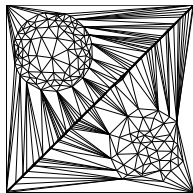
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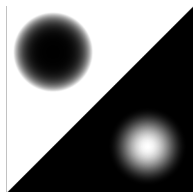


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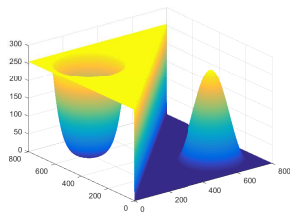
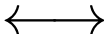


triangulation

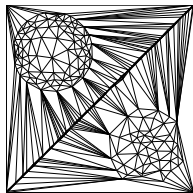
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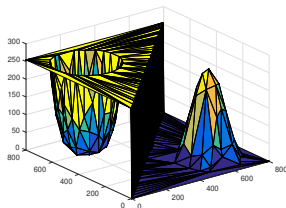
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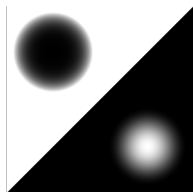


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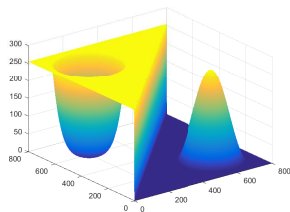
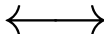


triangle-mesh model

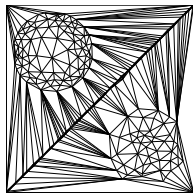
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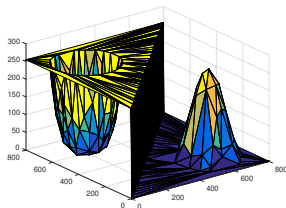
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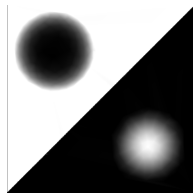
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reconst. image  $\phi'$

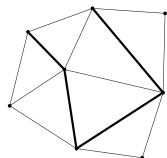


# ERD Mesh Model

- Explicit representation of discontinuities (ERD)
- Piecewise-linear interpolating function
- Based on constrained Delaunay triangulation (CDT)
- Using **wedges** and **wedge values**

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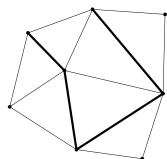
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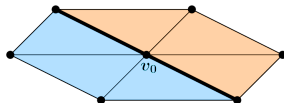
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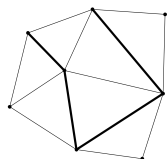
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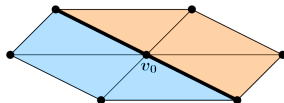
two wedges

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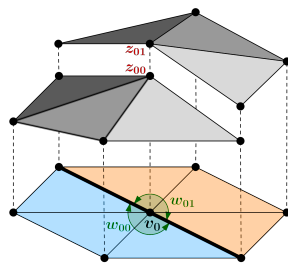
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CDT



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discontinuity modeling

- Process to select model parameters is called **mesh generation**
- Inputs to mesh generation:
  - Image  $\phi$  known on discrete domain  $\Lambda$  of size  $W \times H$
  - $N$  desired number of sample points
- Outputs are ERD mesh model parameters:
  - 1 Set of **sample points**,  $P = \{v_i\}$  (where  $|P| = N$ )
  - 2 Set of **edge constraints**,  $E$
  - 3 Set of integer **wedge values**,  $Z$
- Sampling density of mesh,  $d = \frac{N}{W \times H} \times 100$

# General Steps of ERD Mesh Generation

For selecting model parameters  $(P, E, Z)$  with  $N$  samples

- 1 Initial triangulation:
  - Edge detection
  - Polyline generation + simplification
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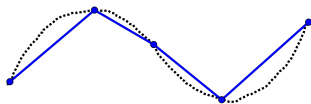


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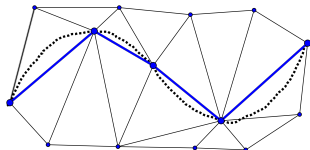


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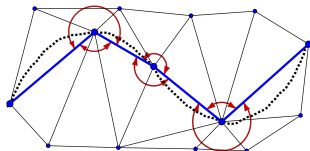




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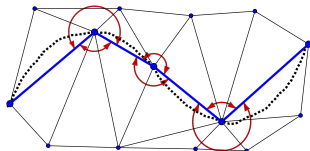
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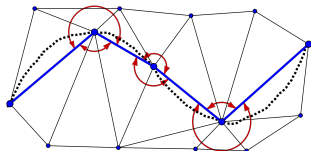
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## 4 Repeat step 3 until $|P| = N$

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- Analyzed previous methods (ERDED and ERDGPI)
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  - 2 Wedge-value selection → optimization-based approach is proposed ⇒ more effective **parameter  $Z$**
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- Combined all modifications ⇒ **proposed SEMMG method**

# Evaluation of SEMMG Method

## ① Compared to ED, MGH, ERDED, ERDGPI methods:

- Test data of 35 images
- 10 sampling densities from 0.0078125% to 3%
- Total of **350 test cases**

	ED	MGH	ERDED	ERDGPI
SEMMG outperforms (% of cases)	100%	89%	99%	85%
Average PSNR increase	8.86 dB	2.25 dB	5.43 dB	2.22 dB

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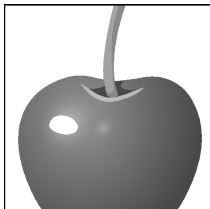
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## ② Compared to GVS, HWT, BSP, ATM methods:

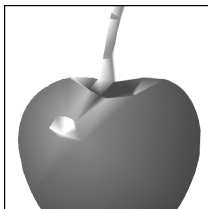
- Average PSNR increase **from 1.10 dB to 3.85 dB**
- **65-80% fewer vertices** compared to GVS method
- **10-60% fewer triangles** compared to BSP method



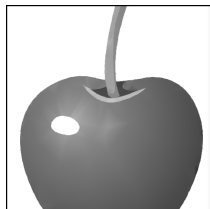
# Visual Examples



original



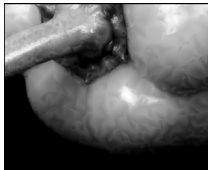
MGH 28.10 dB  
 $d=0.03125\%$



SEMMG 38.78 dB  
 $d=0.03125\%$



original



ERDGPI 37.57 dB  
 $d=0.25\%$



SEMMG 40.70 dB  
 $d=0.25\%$

# Application of Mesh Models in Image Scaling

## Problem Statement

- Common distortions: edge **blurring/ringing** → poor subjective quality



LR image



bilinear with  $k = 4$



bicubic with  $k = 4$

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bilinear with  $k = 4$



bicubic with  $k = 4$

- Goal:** Mesh-based method for producing scaled images with better subjective quality and minimal edge blurring
- Outcome:** MIS method is proposed for scaling grayscale images that are approximately piecewise smooth

# Development of Proposed MIS Method

- General steps of MIS method to scale an image:
  - ① raster image → mesh generation → mesh model of image
  - ② mesh model → mesh transformation → scaled model
  - ③ scaled model → model rasterization → scaled image

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- SEMMG method not designed for image scaling
- SEMMG was used in step 1 to detect **distortions/shortcomings** in image scaling
- Specific **modifications** applied to reduce/eliminate distortions

# Development of Proposed MIS Method Cont'd

- Applied **modifications** to 4 main areas:
  - ① Wedge-value selection: backfilling-based technique  $\Rightarrow$  **more effective parameter  $Z$**
  - ② Mesh refinement: modified centroid-based approach  $\Rightarrow$  **more effective parameter  $P$**
  - ③ Model rasterization: subdivision-based approach  $\Rightarrow$  **smoother edge contours**
  - ④ Polyline simplification: adaptive polyline simplification (APS) technique  $\Rightarrow$  **more effective parameter  $E$**
- Combined all modifications  $\Rightarrow$  **proposed MIS method**

- Experimental Comparisons:

- Methods with **available** implementation
- MIS method is compared to bilinear, bicubic, DCCI, NEDI, and SRCNN methods
- **Subjective Evaluation**
- Objective Evaluations: PSNR, SSIM, PEE metrics

- Conceptual Comparisons:

- Mesh-based methods with **unavailable** implementation
- Differences/similarities using theoretical analysis

# Evaluation of MIS Method Cont'd

## Subjective Evaluation

- Between bilinear, bicubic, DCCI, NEDI, SRCNN, and MIS methods
- 20 LR images and  $k = 4 \rightarrow 20$  HR images with 19 human subjects  $\Rightarrow$  380 rankings
- 300 pairwise comparisons per subject
- Methods ranked from 1st (best) to 6th (worst)



# Evaluation of MIS Method Cont'd

## Subjective Evaluation

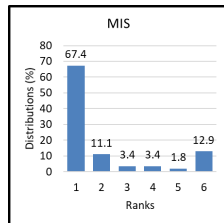
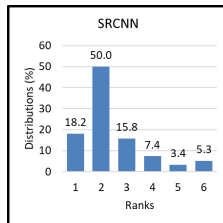
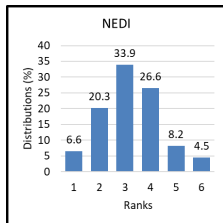
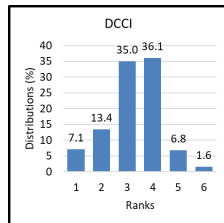
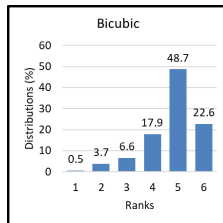
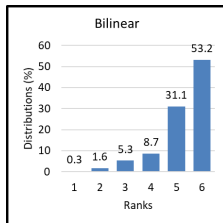
- Between bilinear, bicubic, DCCI, NEDI, SRCNN, and **MIS** methods
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- 300 pairwise comparisons per subject
- Methods ranked from 1st (best) to 6th (worst)
- Statistical properties of the 380 ranks:

	Bilinear	Bicubic	DCCI	NEDI	SRCNN	MIS
Mean Rank	5.28	4.78	3.27	3.23	2.44	<b>2.00</b>
Median Rank	6	5	3	3	2	<b>1</b>
Standard Deviation	0.97	1.02	1.06	1.19	1.26	1.75

- MIS method achieved best mean rank of 2 and median rank of 1

# Evaluation of MIS Method Cont'd

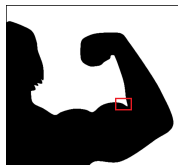
## Subjective Evaluation Cont'd



- MIS method ranked 1st in approximately 67% of cases

# Evaluation of MIS Method Cont'd

## Visual Examples



LR image



LR zoomed in



bilinear



bicubic



DCCI



NEDI



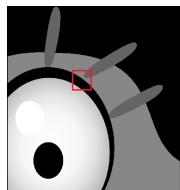
SRCNN



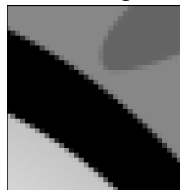
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MIS

## 2 problems were addressed:

- 1 Mesh generation with minimal squared errors
  - The SEMMG method was proposed
  - Improved meshes in terms of both PSNR and subjective quality
  - Compared to ED, MGH, ERDED, and ERDGPI methods **using 350 test cases**:
    - Outperformed ED in 100% with average PSNR margin of 8.86 dB
    - Outperformed MGH in 89% with average PSNR margin of 2.25 dB
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  - Outperformed GVS, HWT, BSP, and ATM methods with average PSNR of 3.85, 0.75, 2, and 1.10 dB
  - **65-80% fewer vertices** compared to GVS method
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- ② Scaling grayscale images with minimal edge blurring using mesh models
  - MIS method was proposed for approximately piecewise-smooth images
  - Improved subjective quality:
    - Sharper and more accurate edges with **minimal blurring/ringing**
    - Compared to bilinear, bicubic, DCCI, NEDI, and SRCNN
    - **Ranked best overall in 67%** out of **380 subjective rankings**
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  - Flexible functionalities:
    - Image models that are **portable, reusable, and editable**
    - Combination of **any affine transformations**: translation, rotation, shearing
    - Almost independent from scale factor



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  - better detection of junction points
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- Color images
  - image model with the same triangulation: scalar  $z$  value  $\rightarrow$  3-tuples  $(r, g, b)$ , **OR**
  - image model with different triangulations: separate triangulation per color channel

# THANK YOU